CHAPTER

6

Probability

Recap Notes

INTRODUCTION

In earlier classes, we have studied the probability as a measure of uncertainty of events in a random experiment. We also learnt the axiomatic theory and classical theory of probability. In this chapter, we shall discuss some more important concepts of probability.

CONDITIONAL PROBABILITY

- The probability of an event A is called the conditional probability of A given that B has already happened.

  $$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Properties of Conditional Probability

- **Property 1 :** The conditional probability of an event A given that B has occurred lies between 0 and 1.
- **Property 2 :** If A and B are any two events of sample space S, and F is an event of S such that $P(F) \neq 0$, then

  $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F) - P((A \cap B) \mid F)$

  In particular, if A and B are disjoint events, then

  $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F)$
- **Property 3 :** $P(A' \mid B) = 1 - P(A \mid B)$

MULTIPLICATION THEOREM ON PROBABILITY

- Let A and B be two events associated with a sample space S. Then $A \cap B$ denotes the event that both A and B have occurred. $A \cap B$ is also written as AB. Sometimes we need to find the probability of the event AB, which is given by

$$P(AB) = P(A \cap B) = \begin{cases} P(A) \cdot P(B|A), \text{ provided } P(A) \neq 0. \\ P(B) \cdot P(A|B), \text{ provided } P(B) \neq 0. \end{cases}$$

  This result is known as multiplication rule of probability.

Extension of Multiplication Theorem

(i) If A, B, C are three events associated with a random experiment, then

$P(ABC)$ or $P(A \cap B \cap C) = P(A) \cdot P(B \mid A) \cdot P(C \mid (A \cap B)) = P(A)\, P(B \mid A)\, P(C \mid AB)$

(ii) If $A_1, A_2, \ldots\ldots, A_n$ are n events associated with a random experiment, then

$P(A_1 \cap A_2 \ldots\ldots \cap A_n) = P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid (A_1 \cap A_2)) \ldots\ldots P(A_n \mid (A_1 \cap A_2 \ldots\ldots \cap A_{n-1}))$.

INDEPENDENT EVENTS

- Two events are called independent if the occurrence or non-occurrence of one does not affect the occurrence of the other.
- Two events A and B associated with a random experiment are said to be independent, if

  $P(A \mid B) = P(A)$, provided $P(B) \neq 0$,

  $P(B \mid A) = P(B)$, provided $P(A) \neq 0$
- Two events A and B associated with a random experiment are independent if and only if

  $P(A \cap B) = P(A) \cdot P(B)$
- If three events A, B, C associated with a random experiment are independent, then

  $P(ABC)$ *i.e.* $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

  and in general, if n events $A_1, A_2, \ldots\ldots, A_n$ associated with a random experiment are independent, then

  $P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1) \cdot P(A_2) \ldots\ldots P(A_n)$

Remarks

(i) Two events A and B are said to be dependent if they are not independent *i.e.*, if $P(A \cap B) \neq P(A) \cdot P(B)$

(ii) Three events A, B, C are said to be mutually independent if $P(A \cap B) = P(A)\, P(B)$, $P(A \cap C) = P(A)\, P(C)$, $P(B \cap C) = P(B)\, P(C)$, and $P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$

If at least one of the above four conditions is not true for three given events, we say that the events are not mutually independent.

(iii) Events A and B are mutually exclusive if and only if $A \cap B = \phi$.

Events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$

(iv) If events A and B having non-zero probabilities are independent, then $P(A \cap B) = P(A) \cdot P(B) \neq 0$. Thus $A \cap B \neq \phi$. Hence, two independent events having non-zero probabilities cannot be mutually exclusive.

BAYES' THEOREM

Partition of a Sample Space

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of sample space S, if

(a) $E_i \cap E_j = \phi$, $i \neq j$, $i, j = 1, 2, 3, \ldots, n$

(b) $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and

(c) $P(E_i) > 0$ for all $i = 1, 2, \ldots, n$

In other words, the events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities.

Theorem of Total Probability

If $E_1, E_2, E_3, \ldots, E_n$ is a partition of the sample space S of a random experiment and A is any event associated with S, then $P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2) + \ldots + P(E_n)\, P(A \mid E_n)$.

Bayes' Theorem

If $E_1, E_2, E_3, \ldots, E_n$ constitute a partition of the sample space S of a random experiment and A is any event associated with the experiment, then

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\, P(A|E_j)}, \text{ where } i = 1, 2, 3, \ldots, n.$$

RANDOM VARIABLES AND ITS PROBABILITY DISTRIBUTIONS

- A random variable is often described as a variable whose values are determined by the outcomes of a random experiment.
- A random variable is called discrete if it assumes only finite or countable number of values. A random variable which can assume non-countably infinite number of values is called non-discrete or continuous random variable.
- Let X is a discrete random variable which can assume values $x_1, x_2, x_3, \ldots, x_n$ (arranged in increasing order of magnitude) with probabilities $p_1, p_2, p_3, \ldots, p_n$.

X	x_1	x_2	...	x_n
$P(X = x)$	p_1	p_2	...	p_n

Then the probability distribution of X is the system of numbers, shown in the table.

Note:

(i) $P(x_k)$ *i.e.*, $P(X = x_k)$ lies between 0 and 1 for $k = 1, 2, \ldots, n$

(ii) $\sum_{i=1}^{n} p_i = 1$

(iii) $P(X \leq x_i) = p_1 + p_2 + \ldots + p_i$,

$P(X \geq x_i) = p_i + p_{i+1} + \ldots + p_n$,

$P(X \leq x_i) = P(X < x_i) + P(X = x_i)$ etc.

Practice Time

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

(a) $\frac{1}{42}$ (b) $\frac{2}{21}$ (c) $\frac{1}{18}$ (d) $\frac{1}{21}$

2. If six cards are selected at random (without replacement) from a standard deck of 52 cards, then what is the probability that there will be no pairs (two cards of same denomination)?

(a) 0.28 (b) 0.562 (c) 0.345 (d) 0.832

3. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find $P(A \cap B)$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

4. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cup B) = \frac{12}{13}$, then evaluate $P(A \mid B)$.

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$ (c) $\frac{9}{13}$ (d) $\frac{4}{5}$

5. If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A' \mid B')$ is equal to

(a) 5/6 (b) 5/7 (c) 25/42 (d) 1

6. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is

(a) $\frac{45}{196}$ (b) $\frac{135}{392}$ (c) $\frac{15}{56}$ (d) $\frac{15}{29}$

7. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

(a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$

8. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is prime number}\}$, find $P(E)$.

(a) 0.87 (b) 0.62 (c) 0.35 (d) 0.50

9. If A and B are two events such that $P(A \mid B) = p$, $P(A) = p$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$, then find the value of p.

(a) 2/3 (b) 4/9 (c) 5/9 (d) 1/3

10. A bag contains 3 white and 6 black balls while another bag contains 6 white and 3 black balls. A bag is selected at random and a ball is drawn. Find the probability that the ball drawn is of white colour.

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

11. Two dice are thrown together. What is the probability that the sum of the number on the two faces is neither 9 nor 11?

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

12. If A and B are two events and $A \neq \phi$, $B \neq \phi$, then

(a) $P(A \mid B) = P(A) \cdot P(B)$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

(c) $P(A \mid B) \cdot P(B \mid A) = 1$

(d) $P(A \mid B) = P(A) \mid P(B)$

13. If A and B are two independent events such that $P(A \cup B) = 0.6$ and $P(A) = 0.2$, then find $P(B)$.

(a) 0.3 (b) 0.4 (c) 0.1 (d) 0.5

14. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by

(a) $1 - P(A)\,P(B)$ (b) $1 - P(A)\,P(B')$

(c) $1 - P(A')\,P(B')$ (d) $1 - P(A')\,P(B)$

15. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

16. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then the probability that both drawn balls are black, is

(a) $\frac{2}{7}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) $\frac{3}{7}$

17. The probability that student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university none will graduate.

(a) 0.216 (b) 0.36 (c) 0.6 (d) 0.1296

18. If two events A and B are such that $P(\bar{A}) = 0.3,\ P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$ then $P(B \mid (A \cup \bar{B})) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

19. Let X denote the number of hours you study on a Sunday. Also it is known that

$$P(X = x) = \begin{cases} 0.2 & , \text{ if } x = 0 \\ kx & , \text{ if } x = 1 \text{ or } 2 \\ k(4 - x), & \text{if } x = 3 \text{ or } 4 \\ 0 & , \text{ otherwise} \end{cases}$$

where k is a constant.

What is the probability that you study atleast two hours?

(a) 0.55 (b) 0.15 (c) 0.6 (d) 0.3

20. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B \mid A) + P(A \mid B)$ equals

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

21. Aprobleminmathematicsisgivento3students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved ?

(a) 1/5 (b) 1/4 (c) 3/4 (d) 2/3

22. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	a	$6a$	$6a$	$4a$	$8a$	$8a$	$6a$	$9a$

Find the value of a.

(a) $\frac{1}{47}$ (b) $\frac{1}{48}$ (c) $\frac{1}{33}$ (d) $\frac{1}{29}$

23. The probability distribution of a discrete random variable X is given below :

X	0	1	2	3
$P(X)$	$\frac{4}{k}$	$\frac{6}{k}$	$\frac{10}{k}$	$\frac{12}{k}$

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

24. The probability distribution of X is

$X = x$	0	1	2	3	4
$P(X = x)$	k	$2k$	$4k$	$2k$	k

Then find $P(X \leq 1)$.

(a) 0.1 (b) 0.3 (c) 0.4 (d) 0.5

25. If A and B are events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(A' \mid B')$ equals

(a) $1 - P(A \mid B)$ (b) $1 - P(A' \mid B)$

(c) $\frac{1 - P(A \cup B)}{P(B')}$ (d) $P(A') \mid P(B')$

26. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

(a) $\frac{33}{56}$ (b) $\frac{9}{64}$ (c) $\frac{1}{14}$ (d) $\frac{3}{28}$

27. You are given that A and B are two events such that $P(B) = \frac{3}{5}, P(A \mid B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A)$ equals

(a) $\frac{3}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

28. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then $P(B' \cap A)$ equals

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$

29. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(a) $\frac{3}{28}$ (b) $\frac{2}{21}$ (c) $\frac{1}{28}$ (d) $\frac{167}{168}$

30. Two events A and B will be independent, if

(a) A and B are mutually exclusive

(b) $P(A' \cap B') = [1 - P(A)]\,[1 - P(B)]$

(c) $P(A) = P(B)$

(d) $P(A) + P(B) = 1$

31. Given that, the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Then find the value of p, if A and B are mutually exclusive.

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{1}{10}$

32. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then find the value of $P(B / A)$.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

33. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then value of $P(A/B)$ is

(a) 0.1 (b) 0.25 (c) 0.5 (d) 0.08

34. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is

(a) $\frac{2}{5}$ (b) $\frac{1}{15}$ (c) $\frac{8}{15}$ (d) $\frac{4}{15}$

35. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$, then $P(A \cup B)$ is equal to

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

36. When the doctor arrives late, what is the probability that he comes by metro?

(a) $\frac{5}{14}$ (b) $\frac{2}{7}$

(c) $\frac{5}{21}$ (d) $\frac{1}{6}$

37. When the doctor arrives late, what is the probability that he comes by cab?

(a) $\frac{4}{21}$ (b) $\frac{1}{7}$

(c) $\frac{5}{14}$ (d) $\frac{2}{21}$

38. When the doctor arrives late, what is the probability that he comes by bike?

(a) $\frac{5}{21}$ (b) $\frac{4}{7}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

39. When the doctor arrives late, what is the probability that he comes by other means of transport?

(a) $\frac{6}{7}$ (b) $\frac{5}{14}$ (c) $\frac{4}{21}$ (d) $\frac{2}{7}$

40. What is the probability that the doctor is late by any means?

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Case II : Read the following passage and answer the questions from 41 to 45.

One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time.

If leap year is considered, then answer the following questions.

41. The probability that it rains on chosen day is

(a) $\frac{1}{366}$ (b) $\frac{1}{73}$ (c) $\frac{1}{60}$ (d) $\frac{1}{61}$

42. The probability that it does not rain on chosen day is

(a) $\frac{1}{366}$ (b) $\frac{5}{366}$

(c) $\frac{360}{366}$ (d) none of these

43. The probability that the weatherman predicts correctly is

(a) $\frac{5}{6}$ (b) $\frac{7}{8}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

44. The probability that it will rain on the chosen day, if weatherman predict rain for that day, is

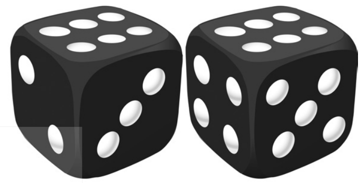
(a) 0.0625 (b) 0.0725 (c) 0.0825 (d) 0.0925

45. The probability that it will not rain on the chosen day, if weatherman predict rain for that day, is

(a) 0.94 (b) 0.84 (c) 0.74 (d) 0.64

Case III : Read the following passage and answer the questions from 46 to 50.

Varun and Isha decided to play with dice to keep themselves busy at home as their schools are closed due to coronavirus pandemic. Varun throw a dice repeatedly until a six is obtained. He denote the number of throws required by X.



46. The probability that $X = 2$ equals

(a) $\frac{1}{6}$ (b) $\frac{5}{6^2}$ (c) $\frac{5}{3^6}$ (d) $\frac{1}{6^3}$

47. The probability that $X = 4$ equals

(a) $\frac{1}{6^4}$ (b) $\frac{1}{6^6}$ (c) $\frac{5^3}{6^4}$ (d) $\frac{5}{6^4}$

48. The probability that $X \geq 2$ equals

(a) $\frac{25}{216}$ (b) $\frac{1}{36}$ (c) $\frac{5}{6}$ (d) $\frac{25}{36}$

49. The value of $P(X \geq 6)$ is

(a) $\frac{5^5}{6^5}$ (b) $1 - \frac{5^3}{6^5}$ (c) $\frac{5^3 \times 61}{6^5}$ (d) $\frac{5^3}{6^4}$

50. The probability that $X > 3$ equals

(a) $\frac{36}{25}$ (b) $\frac{5^2}{6^2}$ (c) $\frac{5}{6}$ (d) $\frac{5^3}{6^3}$

Assertion & Reasoning Based MCQs

Directions (Q.-51 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

51. Let E_1 and E_2 be any two events associated with an experiment, then

Assertion : $P(E_1) + P(E_2) \leq 1$.

Reason : $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$.

52. Consider the system of equations

$ax + by = 0, \; cx + dy = 0$

where $a, b, c, d \in \{0, 1\}$.

Assertion : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

Reason : The probability that the system of equations has a solution is 1.

53. Assertion : Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely.

If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then $P(E|F) = \frac{1}{4}$ and $P(F|E) = \frac{1}{13}$.

Reason : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

54. Assertion : Let E and F be events associated with the sample space S of an experiment. Then, we have $P(S|F) = P(F|F) = 1$.

Reason : If A and B are any two events associated with the sample space S and F is an event associated with S such that $P(F) \neq 0$, then $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$

55. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$.

Assertion : $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Reason : $P(A \cup B) = P(A) + P(B)$.

56. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, ..., n$. Let E be any other event with $0 < P(E) < 1$

Assertion : $P(H_i \mid E) > P(E \mid H_i)P(H_i)$ for $i = 1, 2, ..., n$

Reason : $\sum_{i=1}^{n} P(H_i) = 1$.

57. Assertion : Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Then, the probability that it was drawn from bag II is $\frac{35}{68}$.

Reason : Given, three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, then the probability that the other coin in the box is also of gold is $\frac{1}{2}$.

58. Assertion : An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. Then, the probability that the second ball is red is $\frac{1}{2}$.

Reason : A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Then, the probability that the ball is drawn form the first bag is $\frac{2}{3}$.

59. Assertion : The probability that candidates A and B can solve the problem is $\frac{1}{5}$ and $\frac{2}{5}$, then probability that problem will be solved is given by $\frac{12}{25}$.

Reason : If events A & B are independent, then $P(A \cap B) = P(A) \times P(B)$.

60. A man P speaks truth with probability p and an other man Q speaks truth with probability $2p$.

Assertion : If P and Q contradict each other with probability 1/2, then there are two values of p.

Reason : A quadratic equation with real coefficients has two real roots.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. A bag contains 10 white and 6 black balls. 4 balls are successively drawn out without replacement. What is the probability that they are alternately of different colours?

2. A die is thrown repeatedly until a six comes up. Write the sample space for this experiment.

3. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 yellow and 3 red balls. If it shows head, we throw a die. Write the sample space for this experiment.

4. Find the total number of elementary events associated to the random experiment of throwing three die together.

5. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Write the possible outcomes of this experiment.

6. Two coins (a ₹ 2 coin and a ₹ 5 coin) are tossed once. Find the total number of elements in sample space.

7. Let the sample space associated with an experiment is $S = \{1, 2, 3, 4, 5, 6\}$ and an event is $E = \{1, 3, 5\}$, then find E' or $\bar{E}$.

8. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

9. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

10. If A and B are two events associated with the same random experiment such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, then find $P(B)$.

Short Answer Type Questions (SA-I)

11. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}$ and $\frac{1}{2}$ respectively. Given that $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, find the value of $P(C/B)$ and $P(\bar{A} \cap \bar{C})$.

12. Two thirds of the students in a class are boys and the rest girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

13. Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive events.

14. Find the probability of getting the sum as a prime number when two dice are thrown together.

15. A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Find the total number of possible outcomes for this experiment.

16. What is the probability that all L's come together in the word PARALLEL?

17. A card is drawn from a well shuffled deck of 52 cards, then find the probability of red king card.

18. In a single throw of a die, find the probability of getting an even prime number.

19. If A and B are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find P(not A and not B).

20. A university has to select an examiner from a list of 50 persons, 20 of them are women and 30 men, 10 of them knowing Hindi and 40 not, 15 of them being teachers and the remaining 35 not. What is the probability of the university selecting a Hindi knowing woman teacher?

Short Answer Type Questions (SA-II)

21. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

22. A coin is tossed three times, consider the following events :

A : 'No head appears'

B : 'Exactly one head appears'

C : 'At least two heads appear'

Do they form a set of mutually exclusive and exhaustive events?

23. A bag contains 9 balls of which 4 are red, 3 are blue and 2 are yellow. The balls are similar in shape and size. A ball is drawn at random from the bag. Calculate the probability that it will be:

(i) red (ii) not blue

(iii) either red or blue

24. What is the probability that:

(i) a non-leap year has 53 Tuesdays?

(ii) a leap year has 53 Wednesdays?

(iii) a leap year has 53 Fridays and 53 Saturdays?

25. A shopkeeper sells three types of seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%.

Calculate the probability

(i) that it will not germinate given that the seed is of type A_3.

(ii) of a randomly chosen seed to germinate.

(iii) that it is of type A_2 given that a randomly chosen seed does not germinate.

26. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

27. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that

(i) the youngest is a girl.

(ii) atleast one is a girl.

28. A couple has 2 children. Find the probability that both are boys, if it is known that

(i) atleast one of them is a boy,

(ii) the older child is a boy.

29. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red?

30. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact?

31. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.

32. In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

33. Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

34. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & \text{, if } x=0 \text{ or } 1 \\ 2kx & \text{, if } x=2 \\ k(5-x) & \text{, if } x=3 \text{ or } 4 \\ 0 & \text{, if } x>4 \end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in

(i) exactly one college

(ii) atmost 2 colleges

(iii) atleast 2 colleges.

35. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

Long Answer Type Questions (LA)

36. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so as to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match.

37. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

38. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hosteler?

39. Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

40. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian?

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(d) :** Let D_1, D_2 be the events that we find a defective fuse in the first and second test respectively.

$\therefore$ Required probability $= P(D_1 \cap D_2)$

$$= P(D_1)P(D_2 \mid D_1) = \frac{2}{7}\cdot\frac{1}{6} = \frac{1}{21}$$

2. **(c) :** Let E_i be the event that the first i cards have no pair among them. Then we want to compute $P(E_6)$, which is actually the same as $P(E_1 \cap E_2 \cap ... \cap E_6)$, since $E_6 \subset E_5 \subset ... \subset E_1$, implying that $E_1 \cap E_2 \cap ... \cap E_6 = E_6$.

$$\therefore \quad P(E_1 \cap E_2 \cap ... \cap E_6) = P(E_1)\, P(E_2 \mid E_1)\, P(E_3 \mid E_1 \cap E_2)....$$

$$= \frac{52}{52}\,\frac{48}{51}\,\frac{44}{50}\,\frac{40}{49}\,\frac{36}{48}\,\frac{32}{47} = 0.345$$

3. **(c) :** We have, $S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event that number is even $= \{2, 4, 6\}$

and B be the event that number is red $= \{1, 2, 3\}$

Now, $A \cap B = \{2\}$

$$\therefore \quad P(A \cap B) = \frac{1}{6}$$

4. **(b) :** Given, $P(A) = \frac{7}{13}, P(B) = \frac{9}{13}$ and $P(A \cup B) = \frac{12}{13}$

$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{7}{13} + \frac{9}{13} - \frac{12}{13} \Rightarrow P(A \cap B) = \frac{4}{13}$$

$$\therefore \quad P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{4/13}{9/13} = \frac{4}{9}$$

5. **(b) :** Given, $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}, P(A \cap B) = \frac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{2}{5} + \frac{3}{10} - \frac{1}{5} = \frac{1}{2}$$

$$\therefore \quad P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - \frac{1}{2} = \frac{1}{2}$$

Also, $P(B') = 1 - P(B) = 1 - \frac{3}{10} = \frac{7}{10}$

$\therefore \quad P(A' | B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1/2}{7/10} = \dfrac{5}{7}$

6. (c) : Required probability = $P\{(RBB), (BRB), (BBR)\}$
$= P(RBB) + P(BRB) + P(BBR)$

$= \dfrac{5}{8}\times\dfrac{3}{7}\times\dfrac{2}{6}+\dfrac{3}{8}\times\dfrac{5}{7}\times\dfrac{2}{6}+\dfrac{3}{8}\times\dfrac{2}{7}\times\dfrac{5}{6} = 3\times\dfrac{5}{56}=\dfrac{15}{56}$

7. (d) : We have, $P(A) = 1/4$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\, P(B)$ ($\because$ A, B are independent)
$= 1/4 + P(B) - (1/4)\, P(B) = 2P(B) - 1/4$ (Given)
$\Rightarrow \quad P(B) = 2/5$

8. (b) : $P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$
$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

9. (d) : We have, $P(A)=p, P(B)=\dfrac{1}{3}$ and $P(A \cup B) = \dfrac{5}{9}$

Now, $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = p \Rightarrow P(A\cap B) = \dfrac{p}{3}$

Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \dfrac{5}{9} = p + \dfrac{1}{3} - \dfrac{p}{3} \quad \Rightarrow \quad \dfrac{5-3}{9} = \dfrac{2p}{3} \Rightarrow p = \dfrac{2}{9}\times\dfrac{3}{2} = \dfrac{1}{3}$

10. (d) : Let E_1 = event that bag I is selected
E_2 = event that bag II is selected
E = event that the ball drawn is of white colour
By rule of total probability,
$P(E) = P(E_1)\cdot P(E|E_1) + P(E_2)\cdot P(E|E_2)$

$= \dfrac{1}{2}\cdot\dfrac{3}{9}+\dfrac{1}{2}\cdot\dfrac{6}{9} = \dfrac{9}{18} = \dfrac{1}{2}$

11. (c) : If two dice are thrown, then total number of cases = 36
Number of cases of total of 9 or 11
$= \{(3, 6), (4, 5), (6, 3), (5, 4), (6, 5), (5, 6)\}$

$P(\text{total 9 or 11}) = \dfrac{6}{36} = \dfrac{1}{6}$

P(neither total of 9 nor 11) = 1 – P(total 9 or 11)

$= 1 - \dfrac{1}{6} = \dfrac{5}{6}$

12. (b) : By multiplication theorem,
$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$

$\Rightarrow \quad P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

13. (d) : If A and B are two independent events, then
$P(A\cap B) = P(A)\times P(B)$
It is given that $P(A\cup B) = 0.6$, $P(A) = 0.2$
$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A\cap B)$
$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$
$\Rightarrow \quad 0.6 = 0.2 + P(B)(1 - 0.2)$
$\Rightarrow \quad 0.4 = P(B)\,(0.8)$

$\Rightarrow \quad P(B) = \dfrac{0.4}{0.8} \Rightarrow P(B) = \dfrac{1}{2} = 0.5$

14. (c) : P(atleast one of A and B)
$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\, P(B)$ [$\because$ A, B are independent]
$= P(A) + P(B)\,[1 - P(A)] = [1 - P(A')] + P(B)\, P(A')$
$= 1 - P(A') + P(B)\, P(A') = 1 - P(A')\,[1 - P(B)]$
$= 1 - P(A')\, P(B')$

15. (c) : We have, $\Sigma P(X) = 1$

$\Rightarrow \quad \dfrac{5}{k}+\dfrac{7}{k}+\dfrac{9}{k}+\dfrac{11}{k} = 1 \quad \Rightarrow \quad \dfrac{32}{k} = 1 \Rightarrow k = 32$

16. (d) : Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$.

Now, $P(E) = \dfrac{10}{15}$, $P(F|E) = \dfrac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E).P(F|E) = \dfrac{10}{15}\times\dfrac{9}{14} = \dfrac{3}{7}$

17. (a) : Let X denote the number of students who graduated.
Now, the probability that a student graduates = 0.4
$\therefore$ Probability that a student not graduates = 1 – 0.4 = 0.6
$\therefore$ P (none will graduate) = $(0.6)^3 = 0.216$

18. (d) : $P(\bar{A}) = 0.3 \Rightarrow P(A) = 0.7, P(B) = 0.4 \Rightarrow P(\bar{B}) = 0.6$

$P(A\cap\bar{B}) = 0.5 \Rightarrow P(A\cap B) = P(A) - 0.5 = 0.2$

$P(B|(A\cup\bar{B})) = \dfrac{P[B\cap(A\cup\bar{B})]}{P(A\cup\bar{B})}$

$= \dfrac{P(B\cap A)}{P(A)+P(\bar{B})-P(A\cap\bar{B})} = \dfrac{0.2}{0.7+0.6-0.5} = \dfrac{0.2}{0.8} = \dfrac{1}{4}$

19. (c) : From the given information, we find that the probability distribution of X is

X	0	1	2	3	4
$P(X)$	0.2	k	$2k$	k	0

We know that $\Sigma p_i = 1$
$\Rightarrow \quad 0.2 + k + 2k + k + 0 = 1 \Rightarrow 4k = 1 - 0.2 = 0.8$
$\Rightarrow \quad k = 0.2$
Now, P(you study atleast two hours) = $P(X \geq 2)$
$= P(2) + P(3) + P(4) = 2k + k + 0 = 3k = 3 \times 0.2 = 0.6$

20. (d) : Given, $P(A) = \dfrac{3}{10}, P(B) = \dfrac{2}{5}, P(A\cup B) = \dfrac{3}{5}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \dfrac{3}{10}+\dfrac{2}{5}-\dfrac{3}{5} = \dfrac{3+4-6}{10} = \dfrac{1}{10}$

Now, $P(B|A)+P(A|B) = \dfrac{P(A\cap B)}{P(A)}+\dfrac{P(A\cap B)}{P(B)}$

$= \dfrac{1/10}{3/10}+\dfrac{1/10}{2/5} = \dfrac{1}{3}+\dfrac{1}{4} = \dfrac{7}{12}$

21. **(c) :** Let A, B, C be the respective events of solving the problem. Then, $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$.

Clearly A, B, C are independent events and the problem is solved if at least one student solves it.

$\therefore$ Required probability $= P(A \cup B \cup C)$

$=1-P(\bar{A})P(\bar{B})P(\bar{C})$

$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=1-\frac{1}{4}=\frac{3}{4}$

22. **(b) :** We know that, $\Sigma P_i = 1$

$\Rightarrow a + 6a + 6a + 4a + 8a + 8a + 6a + 9a = 1$

$\Rightarrow 48a = 1 \Rightarrow a = \frac{1}{48}$

23. **(c) :** We have $\Sigma P(X) = 1$

$\Rightarrow \frac{4}{k}+\frac{6}{k}+\frac{10}{k}+\frac{12}{k}=1$

$\Rightarrow \frac{32}{k}=1 \quad \Rightarrow \quad k = 32$

24. **(b) :** $\because \sum_{x=0}^{4} P(X=x)=1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$

$\Rightarrow k + 2k + 4k + 2k + k = 1$

$\Rightarrow 10k = 1 \Rightarrow k = 0.1$

$\therefore P(X \le 1) = P(X = 0) + P(X = 1)$

$\Rightarrow P(X \le 1) = k + 2k = 3k = 0.3$

25. **(c) :** By definition, $P(A' \mid B')=\frac{P(A' \cap B')}{P(B')}$

$=\frac{P((A \cup B)')}{P(B')}=\frac{1-P(A \cup B)}{P(B')}$

26. **(d) :** Required probability $= P(DD) = \frac{3}{8}\times\frac{2}{7}=\frac{3}{28}$

27. **(c) :** We have, $P(B)=\frac{3}{5}$, $P(A \mid B)=\frac{1}{2}$, $P(A \cup B)=\frac{4}{5}$

$P(A \cap B)=P(A \mid B)P(B)=\frac{1}{2}\cdot\frac{3}{5}=\frac{3}{10}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{5}=P(A)+\frac{3}{5}-\frac{3}{10} \Rightarrow P(A)=\frac{4}{5}-\frac{3}{10}=\frac{1}{2}$

28. **(d) :** $P(A) = 0.4$, $P(B) = 0.3$, $P(A \cup B) = 0.5$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.3 - 0.5 = 0.2$

Now, $P(B' \cap A) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2 = \frac{1}{5}$

29. **(a) :** Required probability $= P\{GGB, GBG, BGG\}$

$= P(GGB) + P(GBG) + P(BGG)$

$=\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}+\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}+\frac{2}{8}\times\frac{3}{7}\times\frac{2}{6}=\frac{3}{28}$

30. **(b) :** Two events A and B are independent, if $P(A \cap B) = P(A)\,P(B)$

$\therefore P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)\,P(B)$

$= [1 - P(A)][1 - P(B)]$

31. **(d) :** When A and B are mutually exclusive, then $A \cap B = \phi \Rightarrow P(A \cap B)= 0$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0 \Rightarrow p=\frac{3}{5}-\frac{1}{2}=\frac{6-5}{10}=\frac{1}{10}$

32. **(b) :** Given, $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$, $P(A \cup B)=\frac{3}{5}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{3+4-6}{10}=\frac{1}{10}$

Now, $P(B / A)=\frac{P(A \cap B)}{P(A)}=\frac{1/10}{3/10}=\frac{1}{3}$

33. **(b) :** We have, $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$

$\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.2 + 0.4 - 0.5 = 0.1$

Now, $P(A/B) = \frac{P(A \cap B)}{P(B)}=\frac{0.1}{0.4}=\frac{1}{4} = 0.25$

34. **(c) :** Total number of possible outcomes $= {}^6C_2 = 15$

Possible outcomes $= {}^2C_1\,{}^4C_1 = 2 \times 4 = 8$

$\therefore$ Required Probability $= \frac{8}{15}$

35. **(d) :** Given, $P(A) = 0.4$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$.

Clearly, $P(A \cap B) = P(B \mid A)P(A) = 0.6 \times 0.4 = 0.24$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24 = 0.96$

(36- 40) :

Let E be the event that the doctor visit the patient late and let A_1, A_2, A_3, A_4 be the events that the doctor comes by cab, metro, bike and other means of transport respectively.

$P(A_1) = 0.3$, $P(A_2) = 0.2$, $P(A_3) = 0.1$, $P(A_4) = 0.4$

$P(E \mid A_1)$ = Probability that the doctor arriving late when he comes by cab = 0.25

Similarly, $P(E \mid A_2) = 0.3$, $P(E \mid A_3) = 0.35$ and $P(E \mid A_4) = 0.1$

36. **(b) :** $P(A_2 \mid E)$ = Probability that the doctor arriving late and he comes by metro

$=\frac{P(A_2)P(E \mid A_2)}{\Sigma P(A_i)P(E \mid A_i)}$

$=\frac{(0.2)(0.3)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$

$=\frac{0.06}{0.21}=\frac{2}{7}$

37. (c) : $P(A_1|E)$ = Probability that the doctor arriving late and he comes by cab

$$= \frac{P(A_1)P(E|A_1)}{\Sigma P(A_i)P(E|A_i)}$$

$$= \frac{(0.3)(0.25)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$= \frac{0.075}{0.21} = \frac{5}{14}$$

38. (d) : $P(A_3|E)$ = Probability that the doctor arriving late and he comes by bike

$$= \frac{P(A_3)P(E|A_3)}{\Sigma P(A_i)P(E|A_i)}$$

$$= \frac{(0.1)(0.35)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$= \frac{0.035}{0.21} = \frac{1}{6}$$

39. (c) : $P(A_4|E)$ = Probability that the doctor arriving late and he comes by other means of transport

$$= \frac{P(A_4)P(E|A_4)}{\Sigma P(A_i)P(E|A_i)}$$

$$= \frac{(0.4)(0.1)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$= \frac{0.04}{0.21} = \frac{4}{21}$$

40. (a) : Probability that the doctor is late by any means

$$= \frac{2}{7} + \frac{5}{14} + \frac{1}{6} + \frac{4}{21} = 1$$

41. (d) : Since, it rained only 6 days each year, therefore, probability that it rains on chosen day is $\frac{6}{366} = \frac{1}{61}$

42. (c) : The probability that it does not rain on chosen day $= 1 - \frac{1}{61} = \frac{60}{61} = \frac{360}{366}$

43. (c) : It is given that, when it actually rains, the weatherman correctly forecasts rain 80% of the time.

$\therefore$ Required probability $= \frac{80}{100} \times \frac{6}{366} + \frac{363}{366} \times \frac{8}{10} = \frac{4}{5}$

44. (a) : Let A_1 be the event that it rains on chosen day, A_2 be the event that it does not rain on chosen day and E be the event the weatherman predict rain.

Then we have, $P(A_1) = \frac{6}{366}$, $P(A_2) = \frac{360}{366}$,

$P(E \mid A_1) = \frac{8}{10}$ and $P(E \mid A_2) = \frac{2}{10}$

Required probability $= P(A_1 \mid E)$

$$= \frac{P(A_1)\cdot P(E|A_1)}{P(A_1)\cdot P(E|A_1) + P(A_2)\cdot P(E|A_2)}$$

$$= \frac{\frac{6}{366} \times \frac{8}{10}}{\frac{6}{366} \times \frac{8}{10} + \frac{360}{366} \times \frac{2}{10}} = \frac{48}{768} = 0.0625$$

45. (a) : Required probability $= 1 - P(A_1 \mid E)$
$= 1 - 0.0625$
$= 0.9375 \approx 0.94$

46. (b) : $P(X = 2)$ = (Probability of not getting six at first throw) × (Probability of getting six at second throw)

$$= \frac{5}{6} \times \frac{1}{6} = \frac{5}{6^2}$$

47. (c) : $P(X = 4) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5^3}{6^4}$

48. (c) : $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X = 1) = 1 - \frac{1}{6} = \frac{5}{6}$

49. (a) : $P(X \geq 6) = \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6} + \ldots\infty$

$$= \frac{5^5}{6^6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\infty\right] = \frac{5^5}{6^6}\left[\frac{1}{1-\frac{5}{6}}\right] = \left(\frac{5}{6}\right)^5$$

50. (d) : $P(X \geq 4) = \left(\frac{5}{6}\right)^3 . \frac{1}{6} + \left(\frac{5}{6}\right)^4 . \frac{1}{6} + \ldots\ldots$

$$= \frac{5^3}{6^4}\left[1 + \left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 + \ldots\ldots\right]$$

$$= \frac{5^3}{6^4}\left[\frac{1}{1-\left(\frac{5}{6}\right)}\right] = \left(\frac{5^3}{6^4}\right).6 = \frac{5^3}{6^3}$$

51. (d) : Reason is a standard result. It is addition theorem of probabilities. However, the first result is untrue as we can have

$P(E_1) + P(E_2) > 1$

For example, when a dice is rolled once and

E_1 : 'a number < 5' shows up,

E_2 : 'a number > 1' shows up

then $P(E_1) = \frac{4}{6} = \frac{2}{3}$ and also $P(E_2) = \frac{5}{6}$.

Here, $P(E_1) + P(E_2) > 1$.

52. (b) : The given system of equations can be written as $AX = O$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$.

As $a, b, c, d \in \{0, 1\}$, therefore, matrix A can be chosen in $2^4 = 16$ ways.

The given system has a unique solution namely the zero solution (*i.e.* $x = 0, y = 0$) only if $|A| \neq 0$.

i.e. if A is any one of the following matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

$\therefore$ P(system has a unique solution) $= \frac{6}{16} = \frac{3}{8}$.

Thus, the Assertion is correct.

Further, we know that a homogeneous system is always consistent and admits of atleast one solution namely the zero solution.

So, the probability that the system has a solution is 1. Thus Reason is also correct.
However, Reason is not a correct explanation of Assertion.

53. **(a) :** We have, $P(E)=\frac{13}{52}=\frac{1}{4}$ and $P(F)=\frac{4}{52}=\frac{1}{13}$

Also, $E \cap F$ denote the event the card drawn is the ace of spades.

$\therefore\ P(E\cap F)=\frac{1}{52}$

Hence, $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{52}}{\frac{1}{13}}=\frac{1}{4}$

Since, $P(E)=\frac{1}{4}=P(E|F)$, we can say that the occurrence of event F has not affected the probability of occurrence of the event E. We also have,

$$P(F|E)=\frac{P(E\cap F)}{P(E)}=\frac{\frac{1}{52}}{\frac{1}{4}}=\frac{1}{13}=P(F)$$

Again, $P(F)=\frac{1}{13}=P(F|E)$ shows that occurrence of event E has not affected the probability of occurrence of the event F. Thus, E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.
Such events are called independent events.

54. **(b) : Assertion :** We know that,

$$P(S|F)=\frac{P(S\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$$

Also, $P(F|F)=\frac{P(F\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$

Thus, $P(S \mid F) = P(F \mid F) = 1$

Reason : We have,

$$P((A\cup B)|F)=\frac{P[(A\cup B)\cap F]}{P(F)}=\frac{P[(A\cap F)\cup(B\cap F)]}{P(F)}$$

(by distributive law of intersection over union)

$$=\frac{P(A\cap F)+P(B\cap F)-P(A\cap B\cap F)}{P(F)}$$

$$=\frac{P(A\cap F)}{P(F)}+\frac{P(B\cap F)}{P(F)}-\frac{P((A\cap B)\cap F)}{P(F)}$$

$= P(A \mid F) + P(B \mid F) - P((A \cap B) \mid F)$

55. **(c) :** Since, $P(A \cap B) = P(A)P(B)$, therefore, A and B are independent events.

$$\therefore\quad P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B)}{P(B)}=P(A)$$

Similarly, $P(B|A) = P(B)$.
Thus, Assertion is correct.
However, Reason is not correct for independent events.
For example, when a dice is rolled once, then the events
A : 'an even number' shows up
and B : 'a multiple of 3' show up
are independent as

$$P(A)P(B)=\frac{3}{6}\times\frac{2}{6}=\frac{1}{6}=P(A\cap B)$$

($\because A = \{2, 4, 6\}$ and $B = \{3, 6\}$)

But $P(A \cup B) = P(\{2, 3, 4, 6\})$

$=\frac{4}{6}\neq P(A)+P(B)\qquad \left(\because\ P(A)+P(B)=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}\neq\frac{4}{6}\right)$

56. **(d) :** $P(H_i \mid E) > P(E \mid H_i) \times P(H_i)$

$\Rightarrow\ \frac{P(H_i\cap E)}{P(E)}>P(E\cap H_i)\ \Rightarrow\ P(H_i\cap E)(1-P(E))>0$

$\Rightarrow\quad P(H_i \cap E) > 0$

This leads to a contradiction $0 > 0$ if $H_i \cap E = \phi$ for any i.

57. **(c) : Assertion :** Let E_1 be the event of choosing the bag I, E_2 be the event of choosing the bag II and A be the event of drawing a red ball.

Then, $P(E_1)=P(E_2)=\frac{1}{2}$

Also, $P(A|E_1) = P$(drawing a red ball from bag I) $=\frac{3}{7}$

and $P(A|E_2) = P$(drawing a red ball from bag II) $=\frac{5}{11}$

Now, the probability of drawing a ball from bag II, being given that it is red, is $P(E_2|A)$
By using Bayes' theorem, we have

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}}=\frac{35}{68}$$

Reason : Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen, respectively.

Then, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

Also, let A be the event that 'the coin drawn is of gold'

Then, $P(A|E_1) = P$(a gold coin from bag I) $=\frac{2}{2}=1$

$P(A|E_2) = P$(a gold coin from bag II) $= 0$

$P(A|E_3) = P$(a gold coin from bag III) $=\frac{1}{2}$

Now, the probability that the other coin in the box is of gold
= the probability that gold coin is drawn from the box I.
$= P(E_1|A)$
By Bayes' theorem, we know that

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}}=\frac{2}{3}$$

58. (b) : Assertion : The urn contains 5 red and 5 black balls.

Let E_1 : a red ball is drawn in the first attempt and E_2 : a black ball is drawn in the first attempt.

Then, $P(E_1) = P(E_2) = \frac{5}{10} = \frac{1}{2}$

Now, let E : red ball is drawn in second attempt.

Then, $P(E|E_1) = \frac{7}{12}$ and $P(E|E_2) = \frac{5}{12}$

Now, probability of drawing second ball as red is

$P(E) = P(E_1) \cdot P(E \mid E_1) + P(E_2) \cdot P(E|E_2)$

$$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{1}{2}\left(\frac{7}{12} + \frac{5}{12}\right) = \frac{1}{2} \times 1 = \frac{1}{2}$$

Reason : Let E_1 : first bag is selected, E_2 : second bag is selected.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

Now, let E : ball drawn is red.

Then, $P(E|E_1) = P$ (drawing a red ball from first bag)

$$= \frac{4}{8} = \frac{1}{2}$$

$\Rightarrow$ $P(E|E_2) = P$ (drawing a red ball from the second bag)

$$= \frac{2}{8} = \frac{1}{4}$$

$\therefore$ Required probability

$$P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{8}} = \frac{\frac{1}{4}}{\frac{2+1}{8}} = \frac{\frac{1}{4}}{\frac{3}{8}} = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$$

59. (d) : Probability of solving the problem by A & B is

$= 1 - P$ (None of them can solve the problem)

$= 1 - P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A}) \cdot P(\bar{B})$

$= 1 - [1 - P(A)]\,[1 - P(B)]$

$= 1 - \frac{4}{5} \times \frac{3}{5} = \frac{13}{25}$.

60. (c) : Reason is false.

$P(P$ and Q contradict each other)

$= p(1 - 2p) + 2p(1 - p) = 1/2$

$\Rightarrow$ $8p^2 - 6p + 1 = 0.$

$\Rightarrow$ $(2p - 1)(4p - 1) = 0$

$\Rightarrow$ $p = 1/2, 1/4.$

SUBJECTIVE TYPE QUESTIONS

1. Required probability $= P(BWBW) + P(WBWB)$

$$= \frac{6}{16} \cdot \frac{10}{15} \cdot \frac{5}{14} \cdot \frac{9}{13} + \frac{10}{16} \cdot \frac{6}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = \frac{90}{728} = \frac{45}{364}.$$

2. The sample space is

$S = \{6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), ...\}$

It can be observed that infinite number of possibilities occur.

3. Let H and T represent a head and a tail of a coin, respectively.

Also, let red balls are represented by R_1, R_2 and R_3 and yellow balls are represented by Y_1 and Y_2. Then, the sample space,

$S = \{TR_1, TR_2, TR_3, TY_1, TY_2, H1, H2, H3, H4, H5, H6\}$

4. When three dice are tossed together, then the total number of possible outcomes $= 6^3 = 6 \times 6 \times 6 = 216$

5. The sample space for this experiment is

$S = \{RR, RB, BR, BB\}$, where R denotes the red ball and B denotes the black ball.

6. The sample space is S = {HH, HT, TH, TT}

Thus, $n(S) = 4$

7. Given that, $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$

$\therefore$ $\bar{E} = S - E = \{2, 4, 6\}.$

8. The sample space S for selecting three bulbs at random from a lot is given by

S = {DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}

where D indicates a defective bulb and N a non-defective bulb.

9. The sample space S for the given experiment is

$S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}.$

10. Given, $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$

Now, $P(\bar{A}) = \frac{2}{3} \Rightarrow 1 - P(A) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $\frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4} \Rightarrow P(B) = \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = \frac{2}{3}$.

11. We have, $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$,

Also, $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$

$\therefore$ $P(C/B) = \frac{P(C \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

and, $P(\bar{A} \cap \bar{C}) = P(\overline{A \cup C})$

$= 1 - \{P(A) + P(C) - P(A \cap C)\} = 1 - \left(\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right) = \frac{3}{10}$

12. Let E_1, E_2 and A be the events defined as follows:

E_1 = a boy is chosen from the class, E_2 = a girl is chosen from the class and, A = the student gets first class marks.

Then, $P(E_1) = 2/3$, $P(E_2) = 1/3$, $P(A/E_1) = 0.28$ and $P(A/E_2) = 0.25$.

Thus, $P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) = 0.27$

13. We have, $A = \{(1, 1)\}$, $B = \{(1, 2), (2, 1)\}$ and $C = \{(1, 3), (3, 1), (2, 2)\}$

(i) Since A consists of a single sample point, it is a simple event.

(ii) Since both B and C contain more than one sample point, therefore each one of them is a compound event.

(iii) Since $A \cap B = \phi$.

$\therefore$ A and B are mutually exclusive events.

14. Total number of possible outcomes = 36

$\because$ The sum should be prime number *i.e.*, 2, 3, 5, 7, 11.

Sum 2 ≡ (1, 1)

Sum 3 ≡ (1, 2), (2, 1)

Sum 5 ≡ (1, 4), (4, 1), (2, 3), (3, 2)

Sum 7 ≡ (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

Sum 11 ≡ (5, 6), (6, 5)

$\therefore$ Number of favourable outcomes = 15

Hence, required probability $= \frac{15}{36} = \frac{5}{12}$

15. Corresponding to each doublet we get two outcomes.

For example, corresponding to (1, 1) we get (1, 1, H), (1, 1, T).

Thus, total number of possible outcomes = 36 + 6 = 42.

16. Total number of possible outcomes $= \frac{8!}{3!\,2!}$

Total number of favourable outcomes when (3L's consider as one letter) = $\frac{6!}{2!}$

Hence, required probability $= \frac{6!}{8!/3!} = \frac{3}{28}$

17. The outcomes in the sample space S are 52.

$\therefore$ $n(S) = 52$

Let E be the event of getting a red king card

$\because$ Out of 4 kings, 2 are red and 2 are black

$\therefore$ $n(E) = 2$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

18. Let S = {1, 2, 3, 4, 5, 6} be the sample space and E = {2} be event of getting an even prime number.

$\Rightarrow$ $n(S) = 6$ & $n(E) = 1$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{1}{6}$

19. Here $P(A)\cdot P(B) = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12} = P(A \cap B)$

$\Rightarrow$ Events A and B are independent.

$\Rightarrow$ Events $\bar{A}$ and $\bar{B}$ are also independent.

Now $P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

($\because$ $\bar{A}$ and $\bar{B}$ are independent events)

$= (1 - P(A))(1 - P(B))$

$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$

20. Let E_1 : "A woman is selected", E_2 : "A Hindi knowing person is selected" and E_3 : "A teacher is selected", then $P(E_1) = \frac{20}{50} = \frac{2}{5}$, $P(E_2) = \frac{10}{50} = \frac{1}{5}$, and

$P(E_3) = \frac{15}{50} = \frac{3}{10}$

$\therefore$ Required probability $= P(E_1 \cap E_2 \cap E_3)$

$= P(E_1)\,P(E_2)\,P(E_3) = \frac{2}{5}\times\frac{1}{5}\times\frac{3}{10} = \frac{3}{125}$

21. Six employees can be seated in row in six desks in 6! ways. Married couple can occupy adjacent seats in the following 5 ways.

1 – 2, 2 – 3, 3 – 4, 4 – 5, 5 – 6

Also, they can interchange their seats and the remaining 4 seats can be occupied by remaining 4 employees in 4! ways.

$\therefore$ Number of ways in which married couple will have adjacent seats = 5 × 2! × 4!

So, number of ways in which married couple will have non-adjacent seats = 6! – 5 × 2! × 4! = 480

Hence, required probability $= \frac{480}{720} = \frac{2}{3}$

22. If S be the sample space of tossing a coin three times then S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Now, A (Event of getting no head) = {TTT}

B (Event of getting exactly one head) = {HTT, THT, TTH}

C (Event of getting atleast two heads) = {HHT, HHH, HTH, THH}

Clearly, $A \cap B = \phi$, $B \cap C = \phi$ and $A \cap C = \phi$

So, A, B and C are mutually exclusive events.

Also, $A \cup B \cup C = S$

So, A, B and C are exhaustive events.

23. (i) Total number of possible outcomes = 9

Number of red balls = 4

$\therefore$ Number of favourable outcomes = 4

Hence, required probability = 4/9

(ii) Number of balls which are not blue = 4 + 2 = 6

$\therefore$ Number of favourable outcomes = 6

Hence, required probability = 6/9 = 2/3

(iii) Number of balls which are either red or blue = 4 + 3 = 7

$\therefore$ Number of favourable outcomes = 7

Hence, required probability = 7/9

24. (i) Non leap year contains 365 days *i.e.*, 52 weeks + 1 day

52 weeks contain 52 Tuesdays.

53 Tuesdays means the remaining day is a Tuesday.

Total number of possibilities for remaining day = 7

Number of favourable outcomes = 1

$\therefore$ Probability for 53 Tuesdays $= \frac{1}{7}$

(ii) Leap year contains 366 days = 52 weeks + 2 days

52 weeks contain 52 Wednesdays.

53 Wednesdays means that out of 2 remaining days one will be Wednesday.

Total number of possibilities for remaining two days (SM, MT, TW, WTh, ThF, FSat, SatS) = 7

Number of favourable outcomes = 2

Hence, probability of 53 Wednesdays in a leap year $= \frac{2}{7}$

(iii) Total possibilities for remaining two days of leap year (SM, MT, TW, WTh, ThF, FSat, SatS) = 7

Number of favourable outcomes = 1

Hence, required probability $= \frac{1}{7}$.

25. Consider the following events:

E_i = Seed chosen is of type A_i, $i = 1, 2, 3$

A = Seed chosen germinates.

We have, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$ and $P(E_3) = \frac{2}{10}$

$P(A/E_1) = \frac{45}{100}$, $P(A/E_2) = \frac{60}{100}$, $P(A/E_3) = \frac{35}{100}$

(i) Required probability $= P(\bar{A}/E_3) = 1 - P(A/E_3)$

$$= 1 - \frac{35}{100} = \frac{65}{100} = 0.65$$

(ii) Required probability $= P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$

$$= \frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}=\frac{490}{1000} = 0.49$$

(iii) Required probability $= P(E_2/\bar{A})$

$$= \frac{P(E_2\cap\bar{A})}{P(\bar{A})}=\frac{P(E_2)P(\bar{A}/E_2)}{P(\bar{A})}=\frac{P(E_2)(1-P(A/E_2))}{1-P(A)}$$

$$= \frac{\frac{4}{10}\times\left(1-\frac{60}{100}\right)}{1-\frac{49}{100}}=\frac{\frac{4}{10}\times\frac{40}{100}}{\frac{51}{100}}=\frac{16}{51}$$

26. Probability of choosing bag $A = P(A) = \frac{2}{6}=\frac{1}{3}$

Probability of choosing bag $B = P(B) = \frac{4}{6}=\frac{2}{3}$

Let E_1 and E_2 be the events of drawing a red and a black ball from bag A and B respectively.

$\therefore\quad P(E_1) = \frac{6\times4}{^{10}C_2}$ and $P(E_2) = \frac{7\times3}{^{10}C_2}$

$\therefore\quad$ Required probability $= P(A) \times P(E_1) + P(B) \times P(E_2)$

$$=\frac{1}{3}\times\frac{6\times4}{^{10}C_2}+\frac{2}{3}\times\frac{7\times3}{^{10}C_2}=\frac{8}{45}+\frac{14}{45}=\frac{22}{45}$$

27. Let G_i ($i = 1, 2$) and B_i ($i = 1, 2$) denote the i^{th} child is a girl or a boy respectively.

Then sample space is,

$S = \{G_1G_2, G_1B_2, B_1G_2, B_1B_2\}$

Let A be the event that both children are girls, B be the event that the youngest child is a girl and C be the event that at least one of the children is a girl.

Then $A = \{G_1G_2\}$, $B = \{G_1G_2, B_1G_2\}$

and $C = \{B_1G_2, G_1G_2, G_1B_2\}$

$\Rightarrow A\cap B = \{G_1G_2\}$ and $A\cap C = \{G_1G_2\}$

(i) Required probability $= P(A/B)$

$$= \frac{P(A\cap B)}{P(B)}=\frac{1/4}{2/4}=\frac{1}{2}$$

(ii) Required probability $= P(A/C)$

$$= \frac{P(A\cap C)}{P(C)}=\frac{1/4}{3/4}=\frac{1}{3}$$

28. Let $B_i (i = 1, 2)$ and $G_i (i = 1, 2)$ denote the i^{th} child is a boy or a girl respectively.

Then sample space is,

$S = \{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$

Let A be the event that both are boys, B be the event that at least one of them is a boy and C be the event that the older child is a boy.

$A = \{B_1B_2\}$, $B = \{G_1B_2, B_1G_2, B_1B_2\}$

$C = \{B_1B_2, B_1G_2\} \Rightarrow A\cap B = \{B_1B_2\}$ and $A\cap C = \{B_1B_2\}$

(i) Required probability $= P(A/B)$

$$= \frac{P(A\cap B)}{P(B)}=\frac{1/4}{3/4}=\frac{1}{3}$$

(ii) Required probability $= P(A/C)$

$$= \frac{P(A\cap C)}{P(C)}=\frac{1/4}{2/4}=\frac{1}{2}$$

29. Let A be the event of drawing a red ball in first draw and B be the event of drawing a red ball in second draw.

$$\therefore\quad P(A)=\frac{^3C_1}{^{10}C_1}=\frac{3}{10}\quad\Rightarrow\quad P(\bar{A})=\frac{7}{10}$$

Now, $P(B/A)$ = Probability of drawing a red ball in the second draw, when a red ball already has been drawn in the first draw $=\frac{^2C_1}{^9C_1}=\frac{2}{9}$, $P\left(\frac{B}{\bar{A}}\right)=\frac{3}{9}$

Required probability = P(A/B)

$$=\frac{P(B/A)\cdot P(A)}{P(B/A)\cdot P(A)+P(B/\bar{A})\cdot P(\bar{A})}$$

$$=\frac{\frac{2}{9}\times\frac{3}{10}}{\frac{2}{9}\times\frac{3}{10}+\frac{3}{9}\times\frac{7}{10}}=\frac{6}{27}=\frac{2}{9}$$

30. Let E be the event that P speaks truth and F be the event that Q speaks truth. Then, E and F are independent events such that $P(E)=\frac{70}{100}=\frac{7}{10}$ and $P(F)=\frac{80}{100}=\frac{4}{5}$

P and Q will agree to each other in stating the same fact in the following mutually exclusive ways:

(I) P speaks truth and Q speaks truth *i.e.* $E\cap F$

(II) P tells a lie and Q tells a lie *i.e.* $\bar{E}\cap\bar{F}$.

$\therefore\quad P(P$ and Q agree to each other)

$= P(E)\,P(F) + P(\bar{E})\,P(\bar{F}) = \frac{7}{10} \times \frac{4}{5} + \left(1 - \frac{7}{10}\right)\left(1 - \frac{4}{5}\right)$

$= \frac{28}{50} + \frac{3}{50} = \frac{31}{50} = \frac{62}{100}.$

Hence, in 62% of the cases P and Q are likely to agree in stating the same fact.

31. Let E be the event that A is coming in time; $P(E) = \frac{3}{7}$

and F be the event that B is coming in time, $P(F) = \frac{5}{7}$

Also E and F are given to be independent events.

$\therefore$ Probability of only one of them coming to the school in time $= P(E)\cdot P(\bar{F}) + P(\bar{E})\cdot P(F)$

$$= \frac{3}{7}\cdot\left(1 - \frac{5}{7}\right) + \left(1 - \frac{3}{7}\right)\cdot\frac{5}{7}$$

$$= \frac{3}{7}\cdot\frac{2}{7} + \frac{4}{7}\cdot\frac{5}{7} = \frac{26}{49}$$

32. Let E_1 be the event of getting ghee from shop X, E_2 be the event of getting ghee from shop Y and A be the event of getting ghee of type B.

$\therefore \quad P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2},\ P(A \mid E_1) = \frac{40}{70} = \frac{4}{7},$

$P(A \mid E_2) = \frac{60}{110} = \frac{6}{11}$

Using Bayes' Theorem, we have

$$P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$\Rightarrow \quad P(E_2 \mid A) = \frac{\frac{1}{2} \times \frac{6}{11}}{\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{11}}$$

$$= \frac{\frac{6}{11}}{\frac{4}{7} + \frac{6}{11}} = \frac{42}{44 + 42} = \frac{42}{86} = \frac{21}{43}$$

33. Let I be the event that changes take place to improve profits.

Probability of selection of A, $P(A) = \frac{1}{7}$

Probability of selection of B, $P(B) = \frac{2}{7}$

Probability of selection of C, $P(C) = \frac{4}{7}$

Probability that A does not introduce changes, $P(\bar{I}/A) = 1 - 0.8 = 0.2$

Probability that B does not introduce changes, $P(\bar{I}/B) = 1 - 0.5 = 0.5$

Probability that C does not introduce changes, $P(\bar{I}/C) = 1 - 0.3 = 0.7$

So, required probability $= P(C/\bar{I})$

$$= \frac{P(C)P(\bar{I}/C)}{P(A)P(\bar{I}/A) + P(B)P(\bar{I}/B) + P(C)P(\bar{I}/C)}$$

$$= \frac{\frac{4}{7} \times 0.7}{\frac{1}{7} \times 0.2 + \frac{2}{7} \times 0.5 + \frac{4}{7} \times 0.7} = 0.7$$

34. The probability distribution of X is

X	0	1	2	3	4
$P(X)$	0	k	$4k$	$2k$	k

The given distribution is a probability distribution.

$$\therefore \quad \sum_{i=0}^{4} p_i = 1$$

$\Rightarrow \quad 0 + k + 4k + 2k + k = 1 \Rightarrow 8k = 1$

$\Rightarrow \quad k = \frac{1}{8} = 0.125$

(i) P (getting admission in exactly one college)

$= P(X = 1) = k = 0.125$

(ii) P (getting admission in atmost 2 colleges)

$= P(X \le 2) = 0 + k + 4k = 5k = 0.625$

(iii) P (getting admission in atleast 2 colleges)

$= P(X \ge 2) = 4k + 2k + k = 7k = 0.875$

35. Consider the following events.

E : Two balls drawn are white

A : There are 2 white balls in the bag

B : There are 3 white balls in the bag

C : There are 4 white balls in the bag

$P(A) = P(B) = P(C) = \frac{1}{3}$

$P(E/A) = \frac{^2C_2}{^4C_2} = \frac{1}{6},\quad P(E/B) = \frac{^3C_2}{^4C_2} = \frac{3}{6} = \frac{1}{2}$

$P(E/C) = \frac{^4C_2}{^4C_2} = 1$

$$\therefore \quad P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1} = \frac{3}{5}$$

36. Probability of getting a six by the captains of both the teams A and B is

$P(A) = \frac{1}{6} = P(B)$

$\therefore \quad P(\bar{A}) = P(\bar{B}) = 1 - \frac{1}{6} = \frac{5}{6}$

Since A starts the game, he can throw a six in the following mutually exclusive ways :

$(A), (\bar{A}\bar{B}A), (\bar{A}\bar{B}\bar{A}\bar{B}A), \ldots$

Probability that A wins

$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + \ldots$

$= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$

$$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \ldots$$

$$= \frac{1}{6} + \frac{1}{6}\cdot\left(\frac{5}{6}\right)^2 + \frac{1}{6}\cdot\left(\frac{5}{6}\right)^4 + \ldots$$

This is an infinite G.P., with $a = \frac{1}{6}$ and $r = \left(\frac{5}{6}\right)^2$

Hence the probability of the team A winning the match

$$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

Since the total probability is unity, the probability of team B winning the match $= 1 - \frac{6}{11} = \frac{5}{11}$.

37. Let A, E_1 and E_2 respectively be the events that a person has a heart attack, the selected person followed the course of yoga and meditation and the person adopted the drug prescription.

$$P(A) = \frac{40}{100} = 0.40,\ P(E_1) = P(E_2) = \frac{1}{2}$$

$P(A/E_1) = 0.40 \times 0.70 = 0.28,$

$P(A/E_2) = 0.40 \times 0.75 = 0.30$

Probability that the patient suffering from heart attack followed the course of meditation and yoga is

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28 + \frac{1}{2}\times 0.30} = \frac{0.14}{0.14 + 0.15} = \frac{14}{29}$$

Now, $P(E_2/A) = \dfrac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$

$$= \frac{\frac{1}{2}\times 0.30}{\frac{1}{2}\times 0.28 + \frac{1}{2}\times 0.30} = \frac{0.15}{0.14 + 0.15} = \frac{15}{29}$$

38. Let E_1, E_2 and S be the following events :

E_1 : The student resides in hostel

E_2 : The student is a day-scholar

S : The student attains A grade.

$$P(E_1) = \frac{60}{100};\ P(E_2) = \frac{40}{100}$$

$$P(S/E_1) = \frac{30}{100};\ P(S/E_2) = \frac{20}{100}$$

$\therefore$ Required probability

$$= P(E_1/S) = \frac{P(E_1)\cdot P(S/E_1)}{P(E_1)\cdot P(S/E_1) + P(E_2)\cdot P(S/E_2)}$$

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\frac{60}{100}\times\frac{30}{100} + \frac{40}{100}\times\frac{20}{100}} = \frac{6\times 3}{6\times 3 + 4\times 2} = \frac{18}{18+8} = \frac{9}{13}$$

39. The sample space S of the given random experiment is

$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

Let A be the event that the die shows a number greater than 4 and B be the event that there is at least one tail.

$\therefore\quad A = \{(T, 5), (T, 6)\}$

and $B = \{(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6), (H, T)\}$

$A \cap B = \{(T, 5), (T, 6)\}$

$\therefore\quad P(B) = P\{(T, 1)\} + P\{(T, 2)\} + P\{(T, 3)\}$
$+ P\{(T, 4)\} + P\{(T, 5)\} + P\{(T, 6)\} + P\{(H, T)\}$

$$= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{4} = \frac{3}{4}$$

$$P(A \cap B) = P\{(T,5)\} + P\{(T,6)\} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

$\therefore$ Required probability $= P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{1/6}{3/4} = \frac{2}{9}$$

40. Let A, B, C and E are respectively the events that a person is smoker and non-vegetarian, smoker and vegetarian, non-smoker and vegetarian, and the selected person is suffering from the disease.

Here, $n(A) = 160$, $n(B) = 100$,

$n(C) = 400 - (160 + 100) = 140.$

Also, $P(A) = \dfrac{160}{400}$, $P(B) = \dfrac{100}{400}$, $P(C) = \dfrac{140}{400}$

and $P(E/A) = \dfrac{35}{100}$, $P(E/B) = \dfrac{20}{100}$, $P(E/C) = \dfrac{10}{100}$

$\therefore$ Required probability

$$= P(A/E) = \frac{P(A)\cdot P(E/A)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100} + \frac{100}{400}\times\frac{20}{100} + \frac{140}{400}\times\frac{10}{100}}$$

$$= \frac{5600}{5600 + 2000 + 1400} = \frac{5600}{9000} = \frac{28}{45}$$